Mock Exam on WMPH17001
"Physics of Lasers"

Question 1: Laser Principles (13.5 points)

a) List the three main components of a laser. (1.5 pt)

Pump source
Active medium
Resonator ("two mirrors" is not valid!)

b) State two criteria that distinguish spontaneous emission and laser emission from a suitably pumped material. (1 pt)

Coherence
Directionality
High intensity
Monochromaticity
Threshold behaviour

pick two

c) Name two pump mechanisms. (1 pt)

Optical
Electrical
Direct electrical
Chemical
Gas dynamic

pick two

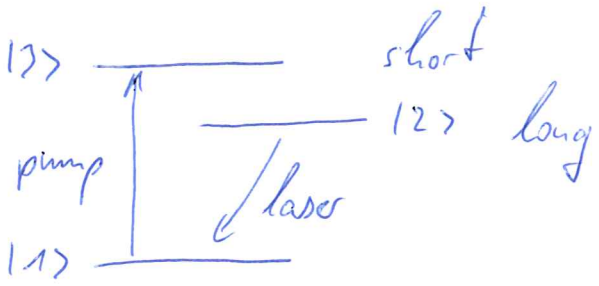
d) State one criterion that a suitable pump source has to fulfil. (1 pt)

Good spectral overlap with pump band of medium
Variation of pump power → pump above threshold
Constant output power → no relaxation oscillations

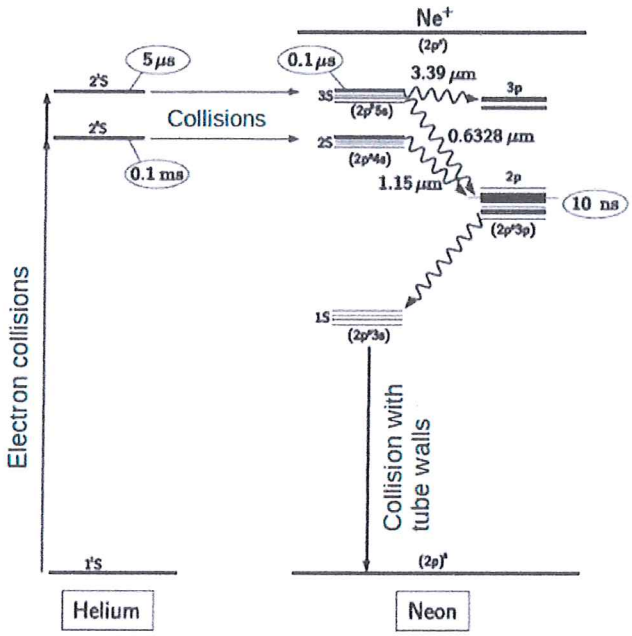
pick one

e) Consider a 3-level laser:

1. Draw the energy level scheme (1 pt).
2. Label the pump transition and the laser transition in each sketch. (1+1 pt)
3. Indicate the requirement for the lifetime of the levels involved in the laser transition to achieve stationary laser operation ('long', 'short lifetime'). (1+1 pt)



f) Consider the following energy level diagram for a Helium-Neon-laser:



1. Is the HeNe-laser a 3- or 4-level laser? Justify your answer briefly. (2 pt)

4-level laser → ground state ≠ lower laser level

2. State one reason why Neon is a good active material for laser action. (1 pt)

- Longer lifetime of upper laser level compared to lower laser level

pick one

- efficient pumping by collisions due to resonance between pump levels in He and upper laser level in Ne

Question 2: Light-Matter Interaction (7 points)

a) Consider a 2-level material with an energy level separation of ΔE. Calculate the relative population of the excited state in thermal equilibrium at room temperature, if the transition energy between the levels is in the visible spectral range. (2 pt)

Vis. range $\Delta E \sim 2eV$; $k_B T \simeq 0.025\,eV$ @ $T = 300\,K$

$$\Rightarrow \frac{n_{exc}}{n_{ground}} = \exp\left(-\frac{\Delta E}{k_B T}\right) \approx 2 \cdot 10^{-35}$$

b) How many photons per mode of the light field are required such that stimulated emission dominates (no derivation of expression)? (1 pt)

$n \geq 1$

c) Briefly discuss why population inversion can not be achieved in a 2-level material. (2 pt)

Increasing absorption increases population of excited state until it is equal to ground state population
⇒ equilibrium between absorption & stim. emission
⇒ no net absorption anymore

d) Explain why stimulated emission was first observed in the microwave region and only later in the visible region. (2 pt)

- $n \gg 1$ in microwave region
  $n \ll 1$ in visible region

- Einstein coefficient $A_{21} \propto \nu^3$ for spont. emission
  ⇒ spont. emission dominates in vis range & is loss channel for stimulated emission

Question 3: Continuous-wave lasers (12.5 points)

a) State the expression of the single-trip gain G_0. (1 pt)

$G_0 = \exp(\sigma \Delta n L)$ σ: absorption cross-section, Δn: inversion, L: length medium

b) Derive the threshold condition for the critical inversion ΔN_{thr} based on the reflectivities (R_1 and R_2) of the resonator mirrors, the single-trip gain G_0 and other losses α'. (4 pt)

"Intensity after round trip in resonator"

$$I_1 = I_0 G_0 R_2 G_0 R_1 e^{-2\alpha' L}$$

with gain ≡ loss @ threshold : $I_1 = I_0$

$$\Rightarrow R_1 R_2 e^{2\sigma \Delta n L} e^{-2\alpha' L} = 1$$

$$\Rightarrow \Delta n = \Delta N_{thr} = \frac{1}{2\sigma L}\left[2\alpha' L - \ln(R_1 R_2)\right]$$

c) What is the meaning of ΔN_{thr} for laser operation? (1 pt)

only for $\Delta n \geq \Delta N_{thr}$ lasing possible

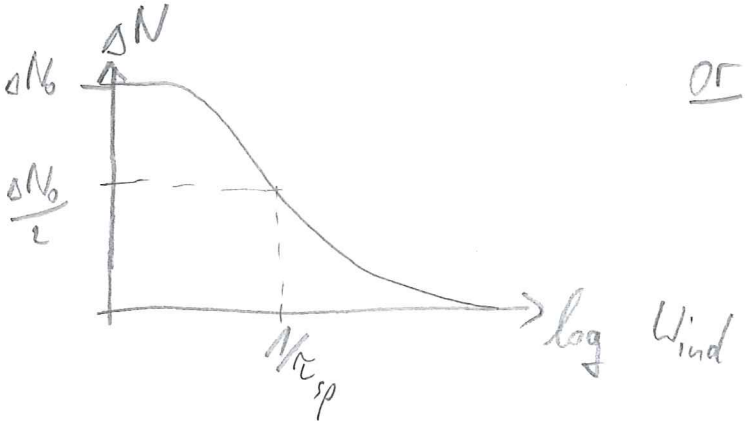
d) Consider a Ruby-laser operating at λ = 694 nm. The Ruby-crystal has a length of L = 5 cm, its laser transition has a cross section $\sigma = 3 \cdot 10^{-20}$ cm^2 and other losses amount to $\alpha' = 0.01$ cm^{-1}. The mirror reflectivities are $R_1 = 1$ and $R_2 = 0.97$. Calculate the critical inversion. (1 pt)

from b): $\Delta N_{thr} = 4.3 \cdot 10^{17}$ cm^{-3}

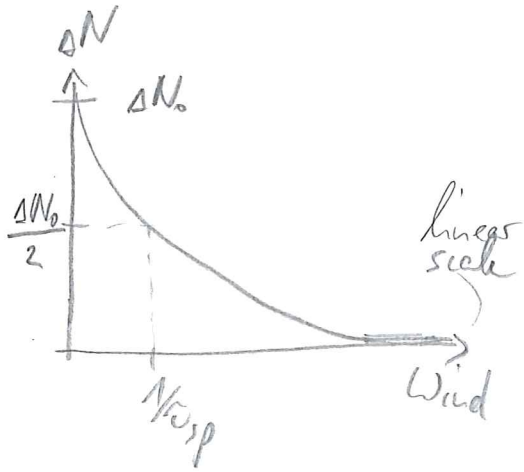
e) Consider a 4-level laser in stationary operation. We derived the following expression relating the stationary inversion ΔN, the rate for the stimulated transition W_{ind} and the spontaneous decay time for the upper laser level τ_{spont}:

$$\Delta N = \frac{\Delta N_0}{1 + \tau_{spont} W_{ind}}$$

1. Qualitatively sketch this function. (2 pt)



or



2. Discuss briefly the limiting cases $W_{ind} \to 0$ and $W_{ind} \to \infty$ (2 pt)

$W_{ind} \to \infty$: equilibrium between absorption and stim. emission

$\Rightarrow \Delta N \to 0$

$W_{ind} \to 0$: spont. emission as only loss channel

$\Rightarrow$ max. stationary inversion ΔN_0 achievable

f) We found in the lecture that ΔN_0 in e) can be expressed as $\Delta N_0 = R_{pump} \cdot \tau_{spont}$, with R_{pump} being the pump rate. For $R_{pump} = 4 \cdot 10^{26}$ cm^{-3}s^{-1} and $\tau_{spont} = 10^{-8}$ s^{-1} calculate the induced rate W_{ind}. (1.5 pt)

from e) $W_{ind} = \frac{1}{\tau_{sp}} \left[\frac{R_{pump} \cdot \tau_{spont}}{\Delta N} - 1 \right]$

$= 8.3 \cdot 10^8 \, s^{-1}$

Question 4: Optical Resonators (13 points)

a) Qualitatively sketch the TEM_{24} and the TEM_{31} transverse modes (in Cartesian coordinates). (2 pt)

TEM_{24} $\qquad$ TEM_{31}



b) What the frequency distance between neighbouring longitudinal resonator modes for a Fabry-Perot resonator with distance d between mirrors. (1 pt)

$$\Delta\nu_M = \frac{c}{2d}$$

c) The gain bandwidth of the red 632 nm line of the Helium-Neon-laser is 1.5 GHz. Assume a Fabry-Perot-type resonator. Determine the distance d between the resonator mirrors so that maximally a single longitudinal resonator mode is within the bandwidth of the gain profile. (2 pt)

$$\Delta\nu_M \geq 1.5\,GHz$$

$$\Rightarrow d \leq \frac{c}{2\cdot\Delta\nu_M} = 0.1\,m$$

d) Assume that you have two mirrors to construct a resonator with distance d between the mirrors. One of the mirrors is curved, the other one is a plane mirror. Under which conditions do you obtain a stable resonator and under which an unstable resonator? (Note: the curved mirror can be convex or concave!) (6 pt)

$R_1 \neq 0$; $R_2 \to \infty$

$$0 \leq \left(1-\frac{d}{R}\right)\left(1-\frac{d}{R}\right) \leq 1 \quad \text{or} \quad 0 \leq g_1 \cdot g_2 \leq 1$$

case 1: $R_1 < 0$: $g_1 > 1$ indep. of d ; $g_2 = 1$ } always unstable

case 2: $R_1 > 0$: $R_1 \geq d \Rightarrow g_1 \leq 1$ and $g_1 \geq 0$; $g_2 = 1$ } stable

e) Consider a Gaussian beam at a wavelength of $\lambda = 500$ nm with a beam waist of $w_0 = 160\ \mu$m and a Rayleigh length $z_0 = 16$ cm.

1. Calculate the angle of divergence θ_0 in the far field (i.e. for $z >> z_0$) for a perfect Gaussian beam. (1 pt)

$\theta_0 = \frac{w_0}{z_0} = 1\,\text{mrad}$ (or $0.057°$)

2. You buy a laser and the manufacturer specifies $M^2 = 1.2$. Which angle of divergence would you actually measure? (1 pt)

$M^2 = \frac{\theta_m}{\theta_0} \Rightarrow \theta_m = M^2 \cdot \theta_0 = 1.2\,\text{mrad}$ (or $0.068°$)